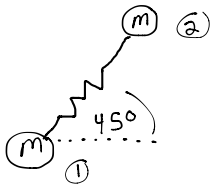
Vibration Structures

Simplest Example:



Find the modes:

$$P = \left[\begin{array}{c|c|c|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & 1 \\ 1 & 0 & -1 & -1 \\ 0 & 1 & 1 & -1 \end{array}\right] \qquad \vec{X} = \begin{bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \end{bmatrix}$$

$\omega^2 = 0$ ← ↓ $\omega^2 = 0$, ↓ $\omega^2 = 0$, → $\omega^2 = \frac{2K}{m}$

new general problem:

$$M = \begin{bmatrix} m_1 & & 0 & \\ & m_1 & & \\ 0 & & m_2 & \\ & & & m_2 \end{bmatrix} \qquad K = \begin{bmatrix}$$

point 2:  , Find $\vec{F}$, and you get K

$$\vec{F} = K(\Delta l)\left(\frac{-\vec{r}}{|\vec{r}|}\right)$$

$\Delta r << \vec{r}$

What is Δl?

$\Delta l = \Delta r \cdot \frac{\vec{r}}{|\vec{r}|}$

$$\vec{F} = -K\frac{\Delta\vec{r}\cdot\vec{r}}{|\vec{r}|^2}\vec{r}$$

Write as matrix?

$$\vec{F} = \begin{bmatrix} F_x \\ F_y \end{bmatrix} \qquad \vec{r} = \begin{bmatrix} r_x \\ r_y \end{bmatrix}$$

$$\begin{bmatrix} F_x \\ F_y \end{bmatrix} = \frac{-K}{|\vec{r}|^2}\begin{bmatrix} r_x r_x & r_x r_y \\ r_y r_x & r_y r_y \end{bmatrix}\begin{bmatrix} \Delta r_x \\ \Delta r_y \end{bmatrix} \quad \ldots\ldots \quad \cos\theta = \frac{r_x}{|\vec{r}|}, \quad \sin\theta = \frac{r_y}{|\vec{r}|}$$

$$= -K\underbrace{\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}}_{-\hat{K}}\begin{bmatrix} \Delta r_x \\ \Delta r_y \end{bmatrix}$$

$$K = \left[\begin{array}{c|c} \hat{K} & -\hat{K} \\ \hline -\hat{K} & \hat{K} \end{array}\right]$$